(1) $\tan x \sin x + \cos x$

$\frac{\sin x}{\cos x} \cdot \sin x + \cos x$

$\frac{\sin^2 x}{\cos x} + \cos x$

$\frac{\sin^2 x + \cos^2 x}{\cos x}$

$\frac{1}{\cos x} = \sec x$ ✓

(2) $\frac{1}{\tan x} + \tan x = \frac{1}{\sin x \cos x}$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$ ✓

(3) $\sin x - \sin x \cos^2 x = \sin^3 x$

$\sin x(1 - \cos^2 x) = \sin x(\sin^2 x)$

$= \sin^3 x$ ✓

(4) $\frac{\cos \alpha}{1+\sin \alpha} + \frac{1+\sin \alpha}{\cos \alpha} = 2\sec \alpha$

$\frac{\cos^2 \alpha + 1 + 2\sin \alpha + \sin^2 \alpha}{(1+\sin \alpha)\cos \alpha} = \frac{2 + 2\sin \alpha}{(1+\sin \alpha)\cos \alpha}$

$= \frac{2(1+\sin \alpha)}{(1+\sin \alpha)\cos \alpha} = \frac{2}{\cos \alpha} = 2\sec x$ ✓

(5) $\frac{\cos \alpha}{1-\sin \alpha} - \frac{\cos \alpha}{1+\sin \alpha} = 2\tan \alpha$

$\cos \alpha + \sin \alpha \cos \alpha - (\cos \alpha - \sin \alpha \cos \alpha)$

$1 - \sin^2 \alpha$

$= \frac{2\sin \alpha \cos \alpha}{\cos^2 \alpha} = \frac{2\sin \alpha \cos \alpha}{\cos \alpha \cos \alpha} = 2\tan \alpha$ ✓

(6) $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

$\dfrac{\csc x \cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \dfrac{\csc x \cos x}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \dfrac{\csc x \cos x}{\frac{1}{\sin x \cos x}}$

$= (\csc x \cos x)(\sin x \cos x)$

$= \dfrac{\cos x}{\sin x} \cdot \sin x \cos x = \cos^2 x$ ✓

(7) $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

$\dfrac{(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{\sin^2 x - \cos^2 x} = 1$ ✓

(8) $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

$\dfrac{\tan^2 x}{\sec^2} = \tan^2 x \cos^2 x = \dfrac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \sin^2 x$ ✓

(9) $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

$\dfrac{1 - \sin x}{\cos x}\left(\dfrac{1 + \sin x}{1 + \sin x}\right) = \dfrac{1 - \sin^2 x}{\cos x (1 + \sin x)} = \dfrac{\cos^2 x}{\cos x (1 + \sin x)} = \dfrac{\cos x}{1 + \sin x}$ ✓

10) $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

$$\frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{\frac{\sin^2 x}{\cos^2 x} - 1}{\frac{\sin^2 x}{\cos^2 x} + 1} = \frac{\frac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}}$$

$$= \frac{(\sin^2 x - \cos^2 x)}{\cos^2 x} \cdot \frac{\cos^2 x}{(\sin^2 x + \cos^2 x)} = \sin^2 x - \cos^2 x$$

$$= (1 - \cos^2 x) - \cos^2 x = 1 - 2\cos^2 x \checkmark$$

11) $\tan^2\theta = \csc^2\theta \tan^2\theta - 1$

$$\rightarrow \csc^2\theta \tan^2\theta - 1$$

$$= \frac{1}{\sin^2\theta} \frac{\sin^2\theta}{\cos^2\theta} - 1 = \frac{1}{\cos^2\theta} - 1$$

$$= \frac{1 - \cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta \checkmark$$

12) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

$$\rightarrow \frac{\cos x}{1 - \sin x}\left(\frac{1 + \sin x}{1 + \sin x}\right) = \frac{\cos(1 + \sin x)}{1 - \sin^2 x} = \frac{\cos(1 + \sin x)}{\cos^2 x}$$

$$= \frac{1 + \sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x \checkmark$$

(13) $\frac{\csc\beta}{\sin\beta} - \frac{\cot\beta}{\tan\beta} = 1$

$\frac{\csc\beta}{1} \cdot \frac{1}{\sin\beta} - \frac{\cot\beta}{1} \cdot \frac{1}{\tan\beta} = \frac{1}{\sin\beta} \cdot \frac{1}{\sin\beta} - \frac{1}{\tan\beta} \cdot \frac{1}{\tan\beta}$

$= \frac{1}{\sin^2\beta} - \frac{1}{\tan^2\beta} = \sec^2\beta - \tan^2\beta = 1$ ✓

(14) $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin^2 x - \cos^2 x$

$= (1 - \cos^2 x) - \cos^2 x = 1 - 2\cos^2 x$ ✓

(15) $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

$\sin^2 x - 2\sin x\cos x + \cos^2 x + \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= 2\sin^2 x + 2\cos^2 x = 2(\sin^2 x + \cos^2 x) = 2$ ✓

(16) $\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

$\frac{(\sin x + 3)(\sin x + 1)}{1 - \sin^2 x} = \frac{(\sin x + 3)(\sin x + 1)}{(1 - \sin x)(1 + \sin x)} = \frac{\sin x + 3}{1 - \sin x}$ ✓

(17) $\frac{\cos x}{1-\sin x} - \tan x = \sec x$

$\frac{\cos x}{1-\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - (\sin x - \sin^2 x)}{(1-\sin x)\cos x}$

$= \frac{\cos^2 x - \sin x + \sin^2 x}{(1-\sin x)\cos x} = \frac{1-\sin x}{(1-\sin x)(\cos x)}$

$= \frac{1}{\cos x} = \sec x$ ✓

(18) $\tan^2 x + 1 + \tan x \sec x = \frac{1+\sin x}{\cos^2 x}$

$\frac{\sin^2 x}{\cos^2 x} + 1 + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos^2 x}$

$= \frac{(\sin^2 x + \cos^2 x) + \sin x}{\cos^2 x} = \frac{1+\sin x}{\cos^2 x}$ ✓